Reflections on Consciousness

Mathematical Model Mimicking Consciousness

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Preface: What We Want to Understand.

Consciousness amazes many of us more than life itself. We take it for granted, as something that accompanies, for example, human life, but for many, simply life. It is some kind of a miracle. I admit, this was my feeling too. In many philosophies and near-religious doctrines, consciousness precedes life: for them, in the beginning there is Consciousness, which constructs life. I intended to lift the veil of miracle from the concept of conscious behavior (however, the concept of "life" and the existence of life remains an inexplicable miracle for me).

Medicine/biology (biomedicine) differs from exact sciences, including mathematics, in that when you think that you have found a solution to the problem before you, this is far from the end of solving it.

In exact sciences, we formulate and solve problems that have not yet been solved. We need to find a solution — any solution. We will work to improve this solution going forward.

However, the problems that we need to solve in biomedicine have already been solved by Nature/God. Our task is not to find some solution, however beautiful and elegant it may be, but to find the solution that Nature/God has already created. Therefore, having found the most elegant approach, we have to conduct experiments to check whether it is used by nature. And it's unlikely that this will be the case on the first try. Thus, everything will go in the garbage, and the work will have to start all over again. However, the veil of mystery around the phenomenon under study will have already fallen, even if the solution found does not correspond to the true solution found by nature. And this is precisely our goal.

We are going to show in this book-article that an equivalent of consciousness automatically arises in structures that depend on a very large

number of parameters (degrees of freedom) and have a sufficiently large memory. However, I have not yet said what we mean when we speak of "consciousness." Since the standard perception of consciousness is associated with a person or, under a more liberal interpretation, with a living being, let me note at once that I have no intention to discuss in this text what life is and how it should be defined. For me, it suffices to understand that it is a very complex system that is self-reproducing, dependent on a very large number of parameters, having a memory, and having a finite existence. Any precise definition will not affect the reflections to follow, and we see before us an incredible number of specific examples. But we will begin our discussion of consciousness with the realization that it accompanies at least some living systems, and perhaps all.

I will present a schematic model, based on well-known (but nontrivial) mathematical observations, that successfully imitates some surprising properties of consciousness.

And still, the question arises: what is consciousness? What do I mean when I say this word?

In actuality, when we say the word "consciousness," we often mean conscious actions. And we need to define what a conscious action is. What I mean by a conscious action is an action that entails a clear previously established goal that this action should achieve — a plan of action is conceived in advance, and the action follows this previously conceived plan. Consciousness in my understanding consists of all conscious actions that a given living being is able to perform. Naturally, depending on this multitude of conscious actions, we can talk about different levels of consciousness. But I am not as concerned with that at the moment.

At the same time, I will look in the model situation for a form of reaction that corresponds to our human understanding of conscious behavior.

The definitions of consciousness that we can find in scientific literature or, for example, in ChatGPT or Google, work their way from the end. This is akin to ancient Greeks starting to build mathematics from algebraic geometry or something even fancier. You need to start mathematics from the axioms of elementary geometry and, say, theorems such as the Pythagorean Theorem. We don't understand anything about consciousness, so we need to start studying this subject with something completely elementary. I choose one elementary component of consciousness — a conscious action. This sounds philosophical, so I will start with an example of a conscious action of a certain bird. I specifically chose an animal — a bird rather than a person — because consciousness is often associated with human activity, with our human mind. I do not accept this, and I want to give an example of animal consciousness that will be very close to our human understanding of consciousness.

This is an episode I observed a couple of years ago. I was sitting in a parked car waiting for my wife Luda. In the middle of the street there was a median with grass and palm trees about every 10 meters. A smallish (but not small) bird flew onto this grass with a large piece of dry bread, a crust. It was an Israeli magpie.



Israeli Magpie. Photo by Udi Steinwell.

It was sitting on the grass, breaking off pieces and eating them. Another similar bird flew up. It looked like they knew each other. The owner of the crust was clearly not going to share. The owner was letting the second bird watch it eat. It was also holding the crust to its side, not allowing the other bird to stand in front of the crust. After standing for a while, the second bird did not take any aggressive steps. It suddenly flew off and sat behind a palm tree about five meters away. It began to intensively dig something there but was doing it in a way that did not let the first bird see what it was digging, just its tail and the legs intensively tossing the soil. The first bird was clearly intrigued — it stopped breaking off pieces of the crust and watched what the second bird was doing. Failing to figure it out, curiosity got the best of it — abandoning the crust, it flew over to join its mate behind the palm tree and have a look. As soon as it landed there, the digging bird took off, grabbed the abandoned crust, and flew away with it to another place (deceived!).

To sum it up, we have observed a conscious action of this second bird. It had a goal (to take possession of the crust) and an action plan to achieve this goal. The plan was successful, it worked.

In the book titled *Life* = *Consciousness*, I described various episodes of my encounter with the animal kingdom, in which it was obvious to me that the animals demonstrated conscious behavior. It was like peeping through a crack into another room, another world, another civilization. Although more often, much more often, this other world showed us that it understands us and knows what we want to see.

Then again, surprisingly, even further out, I have come across trees that I understand a little — trees that apparently exhibit the ability to "think."

One tree species, in the Amazon jungle, knew how to "walk" (moved about five meters over the period of a year; the purpose of this movement was clear). Another species of huge trees built itself based on other trees, which it ate. This was in Cambodia. But why this was done, like some of its other actions, was, let us say, incomprehensible to me. However, even more surprising is the recently discovered behavior of cells of a multicellular living being. Miracles were discovered recently in their life, and not to see consciousness in their behavior is just impossible!

These examples are also described in the book *Life* = *Consciousness*. I will give several of these examples in Appendix A to this text in order to familiarize the reader with widely varying forms of consciousness, conscious actions. I will select examples from the animal world, the plant world, and the cellular world.

However, we need a warning sign here! I wrote about conscious action very confidently. However, perhaps, I am assessing it from the point of view of our human civilization, whereas birds (and other life systems) see and perceive it completely differently. This question mark should be everywhere. We could be misunderstanding the goal, the priorities of a particular animal.

It WAS my conclusion that consciousness accompanies any life, that any life must have consciousness, that life virtually equals consciousness. I wish to emphasize that in the book *Life = Consciousness*, I did not study consciousness. I acknowledged its existence and moved toward the conclusion that life and consciousness are in a state of symbiosis. In another book, *Rare Natural Phenomena and Unusual Coincidences*, in Part 3, I tried to understand why life needs consciousness, I looked for the reason why the symbiosis was necessary.

Among the many reasons that seemed to be potential candidates, only one remained. For all the others, I found counterexamples in nature, that is, species that exist in nature without having this characteristic. I determined that this single characteristic is the instinct of self-preservation, the desire to live. Without consciousness, there is no reason why life would not stop functioning (that is, commit suicide) at the slightest difficulty.

However, my understanding has changed. In this text, I reflect on what consciousness is, and the conclusion I come to is that consciousness arises automatically in every highly complex system, perhaps having certain additional characteristics.

To my own amazement, I remembered that in my initial stage of studying mathematics before the age of 30, the main ideas that I came to were precisely that many mathematical systems with a huge number of parameters have automatic patterns. I named one of them (and it is now called), the Concentration Phenomenon, and I named the other, the Spectrum of Functions, defined on some structures with a huge number of variables. In Appendix B, I put some references where a mathematician reader may find a very short explanation of one of these ideas. I also provide there the necessary references for mathematicians for in-depth reading. But non-mathematicians (for whom this text is mainly written) may just ignore this appendix.

By the way, we can observe how consciousness arises when the number of identical creatures increases — for example, from a single ant to a heap of ants, from a single bee to a beehive. Huge communities of the same species living together create their own form of behavior that exhibits all traits of consciousness. It may be controversial to discuss examples of human communities (although this is clearly seen), but let us consider this in ants.

A small number of ants appear to be acting chaotically, while a large ant hill acts in a coordinated and purposeful manner. For example, they close the entrances into the hill from the rain, and they even know when the rain is coming. They close these entrances the day before. They gather food and wage war against other colonies and other ants. I have observed such wars between different species of ants. They are highly intelligent in many ways. Similarly, one could discuss behavior of a beehive. At the level of human society, we are distracted by details (we know what is going on too well), which obscure the bigger picture of behavior. As a result, to recognize traits of collective consciousness in the society is very difficult.

By the way, as of today, there have been discovered not just ant heaps, but ant settlements, practically "countries." The largest such ant supercolony stretches for 6,000 km along the northern part of Italy and the French coast of the Mediterranean, goes around the coast of Spain, and ends in Portugal. Of course, it consists of a huge number of individual ant heaps (so to speak), which have their own females, queen ants. These heaps do not wage war against each other — they visit each other and even send their workers to help their neighbors when necessary. In total, there have been about five or six such incredible formations hundreds of kilometers long discovered on Earth.



An ant supercolony in Europe. Credit: Giraud, T., Pedersen.

To go back to the mathematical interpretation, today I see these mathematical observations mentioned above as mathematical models — in some sense, the most elementary models standing behind the enormously complex phenomenon that we call consciousness and that arises in the

incredibly large (and structurally complex) system that any form of life represents.

In the next chapter I am going to explain in the most elementary form, without resorting formally to mathematics, these mathematical phenomena that bring us, albeit in a very naive form, closer to a model resembling consciousness. In the separate Appendix B, I will provide references to mathematical works that describe the respective phenomena in precise mathematical language. However, I understand that these analogies are still rather philosophical in nature.

Let me describe my own feeling on the comparisons of mathematical models with the forms of conscious behavior I introduce. On the one hand, I think I feel them quite accurately. Yet, on the other hand, it is extremely difficult to find words that would reflect this understanding. We don't have the appropriate language. However, I decided to write this because consciousness needs to be studied and we need to start somewhere. What I had a chance to read on the subject did not, in my understanding, rise to being even the very first step in studying this complex, and still very little understood, process. Thus, in a sense, this text is just a draft. But I firmly understand that until it is finished, the next stage will not come. This has happened to me with many of my professional mathematical works. And hence, I open myself to criticism (berating) and ridicule.

I will conclude this Preface section by definitions and descriptions of Consciousness from two sources, Google and ChatGPT.

Note that these definitions do not break down the complex concept of consciousness into simple elements. They aim to overview the entire huge puzzle of consciousness at one glance. This creates confusion and essentially transforms one set of unclear words into another. This doesn't explain the picture, at least for me. Also, they are geared specifically toward humans and do not explain at all what may serve as consciousness in other forms of life, especially those distant from mammals, such as trees or cells.

From Google:

Consciousness is (different forms of expressing the same):

- 1. The state of being awake and aware of one's surroundings.
- 2. The awareness or perception of something by a person.

3. The fact of awareness by the mind of itself and the world.

"Consciousness emerges from the operations of the brain."

And other examples (still from Google):

Consciousness is the individual awareness of your unique thoughts, memories, feelings, sensations, and environments. Essentially, your consciousness is your awareness of yourself and the world around you.

Consciousness is an unusual phenomenon to study scientifically. It is defined as a subjective, first-person phenomenon, and science is an objective, thirdperson endeavor.

The Cambridge Dictionary defines consciousness as "the state of understanding and realizing something." The Oxford Living Dictionary defines consciousness as "The state of being aware of and responsive to one's surroundings"; "A person's awareness or perception of something"; and "The fact of awareness by the mind of ...".

And now from ChatGPT:

Consciousness is a complex and multi-faceted phenomenon that refers to the state of being aware of and able to think about one's own existence, sensations, thoughts, and surroundings. It involves subjective experiences, self-awareness, and the ability to perceive and process information. There are various theories and perspectives on consciousness, and it remains a topic of active research and philosophical inquiry.

One of the subject related to consciousness is the level of consciousness. Consciousness is not an all-or-nothing phenomenon. There are different levels of consciousness, ranging from basic awareness to higher-order cognitive functions.

It's important to note that while progress has been made in understanding certain aspects of consciousness, a complete and universally accepted explanation remains elusive. The study of consciousness involves contributions from various disciplines, including neuroscience, psychology, philosophy, and cognitive science. P.S. On the day I sent this book to my formatter, I received comments on the draft from one of my very good friends, Professor Roy Wagner, a philosopher at the University of Zurich. He introduced me to a recent theory of consciousness called the Integrated Information Theory of Consciousness (IIT), which was initiated by Giulio Tononi in 2004. (For further reading, one may refer to the later book by Christof Koch and Giulio Tononi, published in 2013, along with other articles by these authors). According to a source from Wikipedia, IIT 'claims that consciousness is identical to a certain kind of information, the realization of which requires physical, not merely functional, integration.' Most of IIT's developers and main proponents are neuroscientists.

IIT primarily focuses on how the cooperation and interaction between neurons may generate consciousness. Admittedly, my understanding of this theory is very preliminary. However, I find it fascinating and believe it could potentially be integrated with the abstract mathematical model presented in this book. The two theories address fundamentally different aspects of consciousness. For instance, the model in this book proposes an abstract framework of consciousness applicable not only to humans and animals but also to other living entities such as trees and cells, starting from the concept of an abstract conscious action. Perhaps IIT could be adapted to encompass the entire spectrum of conscious actions, thereby extending its applicability to all living beings. I'll leave this intriguing possibility for future exploration.

The Mathematical Model of Consciousness: Concentration Phenomenon; Simplest Example.

In this and the next section, I am going to show that a mathematical model of consciousness can be represented using the Concentration Phenomenon. But first I should explain, using some very simple example, what this phenomenon is.

I will explain the idea of the so-called concentration phenomenon using one simple game I came up with.

The game is presented to two players, A and B. Each of them selects at random 50 numbers, which we will denote

x = [x(1), x(2), ..., x(50)] for A, and y = [y(1), y(2), ..., y(50)] for B.

These numbers can be positive or negative, but we require that the sum of their squares equal one, that is, the sum of $x(i)^2$ (over all i) and the sum of $y(i)^2$ (over all i) must each equal 1. (In mathematics, we call x and y points on the sphere S^50; actually, I should write 49 instead of 50, but we will ignore this difference in our notation.) We then calculate the following number: the sum of the product x(i)y(i) (over all i). Let us denote this sum by Z. That is, Z is a function $f\{x(1), x(2), ..., x(50), y(1), y(2), ..., y(50)\}$; a function of a given set of numbers in mathematics is a number that is obtained according to a given rule from these given numbers.

Now, if Z is positive, then A wins, and if Z is negative, then B wins. If Z = 0, then in this case I win. You might be thinking that the game is even too fair. I'll never get anything. However, the computer that calculates the number Z does not calculate it with infinite precision. It must stop after a certain number of calculated decimal places. Let's say it stops after two decimal places. That is, if the number is, for example, 0.438..., the computer will give us the number 0.43.

Perhaps you naively think that the game is still fair and that the number of my wins will be very small. But you are mistaken. More than a quarter of all wins will be mine. And, if instead of 50 numbers, we take 10,000 numbers x(i) and y(i) [where i now varies from one to ten thousand], then my winning would be totally dominant — almost every game would be my win. This fact is completely counterintuitive.

The fact is that with a probability very close to 1 (i.e. almost surely; in our jargon we say "with a very high probability"), when choosing the numbers x(i) and y(i), i = 1, 2, ..., the function Z takes values very close to zero. (This is a fact, which is the result of a specific computation; it is not intuitively obvious.) We say that the Z function concentrates around the value of zero. And the more parameters are involved (that is, the more numbers x(i) and y(i) we take, say 10,000 instead of 50), the greater the likelihood that the result will be very close to zero. And as the number of parameters increases, it will get closer and closer to zero.

Precisely this is the phenomenon of concentration of functions that depend on a very large number of variables. This was not a specific property of the Z function we discussed above. What mattered was that this function was defined on the sphere Sⁿ for large n — in our case, I considered n = 50, and then mentioned n = 10,000. Any "good" (in a certain sense) function on a sphere of large dimension will, with huge probability, keep taking values around the same number.

If we are not interested in extremely unlikely events, then essentially, this function, any "good" function, is almost constant. The realm of such "good" functions to be considered includes, for example, functions that do not oscillate much locally. For example, if two points on a sphere are no more than, say, one unit apart from each other, then the values of our function should differ by, say, no more than ten units. (I'm not being entirely precise here. We call such functions Lipschitz functions (Lip-functions for short) with a constant C, and, in my example above, C=10).

In reality, a much larger function class than Lip-functions will satisfy the same property. Moreover, the concentration property is extremely stable. If the function f we are considering, given on the sphere S^n, takes with a very high probability a value around a number, say, 5, then changing this even with the use of fairly significant modifications (perturbations) is not at all easy. We can take any function g(x), g(0) = 0 (x is a point on the same sphere S^n), |g| < 100 (for example), dependent on not too many variables, that is, for example, g(x(1), x(2), ..., x(sqrt(n))), and not dependent on the rest of x(i), and the perturbed function f + g will still take with great probability

approximately the same value 5. For this, it is only important that the number of parameters n (we call it dimension) is sufficiently large.

The concentration phenomenon I described and its properties are true not only for a sphere, but also for a huge number of objects and systems (say, behavioral systems) if they depend on a very large number of variables. In Appendix B, I provide a reference to a long list of mathematical objects for which these are proven theorems.

Presumably, each task assigned to a certain system (living organism) can be described by some function, not yet known to us today, which depends, naturally, on a very large number of parameters. And if the model example of concentration is also valid in this case, we get behavior close to a fixed pattern. I call this the system's **conscious response**. This reaction does not change when a small number of parameters are changed. **Consciousness must be stable**. And conservative. When we come back to approximately the same situation, we should demonstrate the same reaction.

This property of consciousness (and the concentration model we are considering) is difficult to overestimate. Repeating a reaction allows one to understand (remember) which decision is correct and select (remember) the best decision. However, completely identical situations do not arise, and it is necessary to implement the same action plan in situations that are close, but not identical. And the concept of "closeness" is not a common one. It completely ignores changes, even very significant ones, but in a small number of variables.

In our model example that I presented in the preface, many parameters could have been different. For instance, the size of the piece of bread, the obstacle behind which one of the birds was hiding in order to deceive the other, the weather, and so on and so forth. But all of them would be immaterial to the bird's solution for getting possession of the bread. And its plan would have most likely worked.

Conservative Nature of Consciousness and One Well-Known Casino Rule

I will now give a fictional example of conservative behavior by us, humans.

A person is walking along a road that in one place forks and later converges again. The man can go right or left. He chooses to go right. After some time, he suddenly finds a brilliant diamond in the dust of the road. Several days pass, and the same man is again walking along the same road. We are all quite sure that when he approaches the fork in the road, he will turn right without hesitation. But why? What he found last time is no longer there. On the contrary, perhaps by going left he might find something valuable.

I want to compare this behavior of our consciousness with a wellknown rule of casinos — closing down the table after a big win taking place there.

This decision by the casino seems very strange to mathematicians. It is assumed that the outcome of each game is independent of the previous game and does not affect the future. But then why is it necessary to close down the table where a very big win took place? From the standpoint of a naive layperson, perhaps it would be the opposite — if a win has already occurred, the next one should not be expected for a long time.

But casino owners are not stupid or religious people. They are pragmatic and know from their own experience that it is risky to continue the game at the same table, and they need to close it down for a while. I see an analogy between the behavior of the person in the previous example and the behavior of the casino owners in this example. In a game (or behavior of systems with a huge number of parameters) repeated many times, **wins and losses exhibit the property of stability, and I will try to explain this based on simple rules of concentration.**

Of course, this will be a very naive explanation. But this is the purpose of this text — to present some explanations of the properties of consciousness that we observe, while using mathematical terminology as little as possible. The reference to a more serious explanation with the use of mathematical terminology will be given in the separate Appendix B.

In the previous chapter, to explain the idea of concentration, we chose a sphere of large dimension Sⁿ, that is, our n was large. To explain the stability of wins (successes) and losses (failures), our model will be a deck of cards. We will denote it Π as in permutation. Indeed, with a given ordering of the cards we may associate one permutation of the original card deck; so, ordering means some element of Π which we call x. We have a total of 52 cards in the deck, so we call it Π_52 (we could consider permutations of n different cards, in which case it would be Π_n). We start the game with some x, an ordering of the cards unknown to us, and denote the outcome of the game by f(x). This is our win (if f(x) > 0) or loss (if f(x) < 0). In this setup, the skill of the player is incorporated into the function f.

A fair game means that our function is in a "fair situation," correctly balanced, which is expressed by writing |Ef = 0. This means that the sum of all possible losses and wins is zero. That is, the amount you can win is also the amount you can lose. Now I will explain that this rule is not as fair as it seems at first. For that purpose, we introduce another number, Lf, called the median. This is a number such that the number of initial positions of our deck x with f(x) > Lf equals the number of positions for which f(x) < Lf. (Again, I am not entirely precise here. In a more precise definition, we consider a number Lf such that the number of cases in which $f(x) \ge Lf$ is greater than or equal to $\frac{1}{2}$ of all possible cases, and the same is true for the number of cases where $f(x) \leq Lf$. This adjustment to the previous simple definition is necessary, since it may happen that the number of cases where f(x) = Lf is significant.) If all wins were the same and all losses were also the same, then these two numbers, |E and Lf, would be equal. But wins have different weights, sometimes large, sometimes small, as do losses. And therefore, even when one number is zero, the other number Lf can be positive or negative. However, we know, and it is a mathematical fact (Theorem), that the concentration of our outcomes occurs around the number Lf.

Accordingly, if Lf is positive, we will win more often, and if it is negative, then we will lose more often. Therefore, if Lf > 0, we can win very often (but that very rare loss can be much greater, so that the losses compensate the casino for all the wins). I described this situation more precisely in a mathematical article, a reference to which is provided in Appendix B. Here I will conclude with only general remarks. However, we will also explain why these observations may help explain the emergence of consciousness, as well as the learning process central to the formation of consciousness.

Let me repeat that the above discussion was very preliminary and naive. I wanted to explain that even a small "imbalance" in the function f may lead to dramatic unfairness because of the high dimensionality and strong concentration.

Question: can we "beat" the fate and, with small losses, predict whether or not we have a favorable game?

The concentration estimates are doing it, and with very few events (games) (and even one may be good enough), we may state with a high probability whether this game is in our favor or not. Indeed, if Lf > 0 (a favorable game), i.e., the median Lf is inside the positive region for the function f(x), and it is not too small (say, of the order $1/\sqrt{n}$), then already the probability of the first win is far above 1/2, and after very few events (games), we may be sure with a very high probability that this series is favorable for us. I would like to emphasize that the conclusion of this discussion is opposite to what we, mathematicians, would consider to be the right strategy. We usually tend to accept an absolute, ideal independence of outcome of the next game from the previous one. So, say, three losses in a row will not discourage us from the next attempt (and, perhaps, without admitting it even to ourselves, we would believe in luck on the next attempt even more).

However, the ideas we discussed above suggest stopping play of this series (after three losses) immediately. I would like to end this section by explaining what pushed me to create this mixture of the firm mathematical concept with some philosophy, instead of computing some precise examples. I was always surprised by some well-known rules of casinos (as, say, the closure of a table where significant wins happened; but why?), or by some expressions, such as "a period of luck" (or, conversely, "a period of bad luck"). It sounded like complete nonsense to my mathematical culture and taste. I am not so sure about this now, taking into account the enormously high number of parameters that implies sensitivity of the functions (outcomes) to unknown, very small perturbations, which may lead to dramatic impact on these functions.

I will now provide one experimental confirmation of the situation described. A very good acquaintance of mine is a professional poker player. He plays about thirty games a day on the computer and wins a lot. I decided to ask him if he sometimes senses periods of success and, conversely, periods of failure. His reaction amazed me. It turns out that all players know that this is exactly the case. They even have special terminology.

They call it "**upstreak**" and "**downstreak**." I was even shown graphs of wins and losses. Those are zigzags (naturally), but they often show significant duration in each movement upward (wins) or downward (losses). Then again, smart players stop playing when the trend is downward and return to the game after some time. For that reason, a zigzag going up is often longer than one going down. I consider this an experimental confirmation of the reasoning presented above, since I asked this question to find out whether or not this was the case, without expecting that they knew this. And the answer was confirmation.

In the following section we use the stability of win/loss situations to demonstrate additional analogy between consciousness and our mathematical model based on the Concentration Property of high-parameter systems.

Memory and Consciousness

We will now try to explain how the previous win-loss scheme explains certain traits of consciousness and also the development of reflexes.

But first, we should develop the language to use in discussing our subject.

We explained in the preface what we mean by conscious action. In the section Concentration Phenomenon: Simplest Example, we considered the example of a sphere of very large dimension, that is, depending on a very large number of variables. We saw that sufficiently good functions, which we called Lip-functions, with huge probability take only one value (specific to each of these functions). We proposed to call this value the **element of consciousness** reflected by this function. (We should see such value as a reaction/action of our system). We also noted there that this property is extremely stable. We can change it with not too many variables, and this element of consciousness will not change. Using the sphere S^n as an example, I will state this more precisely.

Let us be given one such function (that is, Lip-function) f. Let us consider the family (= the universe) of all functions of type f + g, where g is any of the Lip-functions (defined on our sphere S^n) that depends on not too large a number of variables, say less than sqrt(n), and also bounded by a number, say 100. We noted in the same section that any of these functions will represent the same element of consciousness, that is, approximately the same number will be accepted by this function with a probability very close to one. Therefore, I will call this entire set of functions derived from one function f a *capsule* of consciousness.

Of course, we can start with another function from this family and build its capsule, and it will be about the same. This concept can be changed in such a way that the capsule will be exactly the same, although this is not significant for us. This is why, from now on, I will talk about capsules of consciousness. Each of them is a family, a very large family of functions that also exhibit the same **element of consciousness**.

We expect to see this property in our understanding of consciousness, as a manifestation of consciousness (the same reaction in a similar but not necessarily identical situation). And we also see this property in the model based on Concentration Phenomenon, already as a result, as a theorem.

Let's now play an imaginary game: the subject, let's call him X, turns on the light, then with a high probability he receives something pleasant, let's call it "y". If X is a person, then one experiment is enough, maybe two, and X will know what to do to get the pleasant "y" — he has developed a reflex. But if it is an animal, say a monkey or a dog, that can turn on this light, then it will also learn to turn it on in order to receive its coveted "y". This is the level of reflex development. This is a consequence of memory. Only memory is needed for this.

Now let us imagine a life situation where the plan for obtaining "y" may turn out to be complex, with a very large number of parameters involved in the relevant action. However, we know that two or three experiments should already show us whether the plan is working. The results are concentrated around winning or losing. If the plan leads to winning, then after two or three experiments (and maybe even one), X knows that this is a good plan, and the plan is written into memory. It becomes part of our knowledge — it has entered our consciousness.

In actuality, the entire capsule of this plan is recorded in memory and this capsule of consciousness transforms into a part of our consciousness. From now on, in other situations that are only similar to the one in which the plan was developed, we will "consciously" choose this plan.

Without memory, consciousness cannot arise. The acquired experience must be preserved. We will try to follow the same plan when we need "y".

One more note. One day in 1999, at the end of his talk, Gromov asked who among those present had Clinton's phone number (who was the U.S. president at the time). Before we had a chance to be astounded, he continued: "How does each of you immediately know that he does not have it?" Indeed, why didn't we start searching for it in our memory?

It looks that we know this right away, a priori. This is a remarkable (and mysterious) property of memory (and consciousness) — to sometimes immediately know that we have absolutely no knowledge of something. Within our framework, this can be explained. Information about lack of knowledge in itself constitutes knowledge — it transforms to become part of

our consciousness. Based on several key words of the question, we immediately look for the corresponding capsule of knowledge and, when we fail to locate it, categorize this lack of knowledge as a new element of our consciousness. Of course, one needs to search through all the capsules, but this is not a problem at all.

For example, there are perhaps a trillion different phone numbers in the world today. And when we call from our phone to any other, it is located so quickly that we do not notice it. There are algorithms for that. They use expanders — a mathematical construct that also involves concentration.

(The conference that was mentioned is "Visions in Mathematics — Towards 2000" that was held at the Tel Aviv University in August 25 – September 3, 1999. All talks were recorded on video and may be found through the home page of the university's School of Mathematics.)

Reducing Oversimplification

In the previous sections we used the power of the concentration method to less than its full potential. I am afraid that even a very interested reader, who is, however, not a mathematician, would not be able to "withstand" the use of this method to its full potential and may have not understood the central idea of using the method of concentration for the model that mimics consciousness. However, on the other hand, it is precisely this simplification that seriously diminishes the perception that the model is consistent with consciousness. Our model is perceived as something far more restricted than what we typically understand as consciousness. I will now try to slightly reduce this oversimplification by explaining some additional properties of the concentration method and putting them into play.

In the section "The Mathematical Model of Consciousness" we discussed the phenomenon of concentration of functions that depend on a very large number of variables. Any "good" function (in a certain sense — for example, Lip-functions) on a sphere Sⁿ of a large dimension n will, with huge probability, keep taking values around the same number. Let us denote, for a function f, this concentration number as C(f).

I should note here that the same effect would be observed if we considered not one function f, but many functions { $f_i(x)$, i = 1, 2, ..., k} simultaneously. In our example of a sphere S^n, x is a point on the sphere, and with a probability very close to 1, all $f_i(x)$, for all i = 1, 2, ..., k, would be approximately equal to the corresponding concentration numbers C(f_i) at the same point x. Of course, I did not say how large the number k can be, i.e. how many functions can be taken at the same time. For a sphere, this number can greatly, by an order of magnitude, exceed the dimension n. I will not elaborate on this here. Let me just note that this is also true for all examples that we consider in mathematics. Again, from the works listed in Appendix B, this number is obvious to a mathematician. We will denote this set of functions { f_i } by **f** and call **f** a vector function. We call the corresponding set of numbers {C(f_i), i = 1, 2, ..., k} the concentration vector of this family and denote it as C(f).

Let us also note that the sphere that we considered as the main model represents the simplest example. Sets X are possible such that "good" functions on them will have more than one possible value as concentration values, for example two or more, or even some set of values that has a certain structure.

For example, it is easy to specify an X for which the statement will read as follows: any "good" function f (that is, for example, a Lip-function) given on X, takes one of two values: $C_1(f)$ or $C_2(f)$ with a probability very close to 1.

In the section "Memory and Consciousness" we introduce the notion of a *capsule* of consciousness. It is the entire family of functions derived from one function f by an "allowed" perturbation. By such an allowed perturbation we consider the family of all functions of type f + g, where g is any of the Lipfunctions that depends on not too large a number of variables, say less than sqrt(n), and is also bounded by a number, say 100. Any function from this family has the same concentration number C(f) and leads to the same response/action (of our system).

However, the central role in our model should be played by the aggregation of all capsules derived from functions corresponding to / describing one specific array of conscious actions of our system, which I will call a "supercapsule."

And, in actuality, the entire super-capsule of the plan of action is recorded in memory, and this super-capsule of consciousness transforms to become part of our consciousness.

There is one more point to be noted here. A conscious action is not necessarily elementary, in the sense that it may contain other conscious actions already created earlier as part of the new plan. For example, in the introduction, I discussed the behavior of a bird that developed a plan to take away a piece of bread from another bird. Part of this plan was to hide its actions behind a tree. Knowing that you can hide your actions behind a tree constitutes another conscious action, which was already one of the conscious actions of this bird, part of its consciousness.

Thoughts by Association and Emergence of New Ideas

What we call a new idea is actually a new element of our consciousness, a new "conscious action" that enriches our consciousness. In this case, I also categorize "understanding" and emergence of "knowledge" as conscious actions.

How does an idea suddenly spring to mind, how do analogies develop? It's hard to grasp. The line of our thought, the sequence of analogies that lead from one thought to another, progresses very quickly and is immediately forgotten. We immediately focus our attention on the result at the end of that chain of thought leaping from one episode to another, and when, even a minute later, we want to understand how the thought came about, the entire path has already "faded," the brain has forgotten about it. Early on in my mathematics studies, I was extremely keen to understand how this happens. To catch the sometimes unexpected transitions from one thought to another. Only a few times did I manage to grab this chain by the tail and unravel it back before it disappeared from memory. The results were astonishing.

The mind progressed from the initial to the final episode — the result that I captured — through six or seven other episodes, each time based on a very clear association. These episodes look like photographic snapshots. We humans generally think in geometric images, and a snapshot like this appears because of some specific image in it. This single image is associated with the previous snapshot. But the next snapshot is already associated with another subject of this snapshot. This is how leaps occur from one situation to another that are contextually very far apart. There was absolutely no connection between the middle links and the end or the beginning. At the same time, the final thought was often important and made sense, whereas the intermediate links were not important (or needed)!

Perhaps these occur in our heads all the time, and in large numbers such short lightning impulses. But we forget them if they don't carry any meaning for us. However, those that do have a meaning stick in the memory as new thoughts, new ideas.

Perhaps one might think that the picture I described is reminiscent of Proust and James Joyce's "stream of consciousness" technique. But I don't think so. Joyce describes the flow of thoughts — what and how we think —

when, simply put, we are doing nothing. These are not disappearing lightning impulses that flash by and then immediately vanish. I am not sure if he knew about those.

In the abstract scheme of consciousness capsules, some consciousness capsule X is activated by the state of the system. A certain representative of this capsule, recorded in our memory by a certain snapshot scene, shifts its emphasis from the central element of the snapshot for this consciousness capsule X to a non-central element, which turns out to be central for some other consciousness capsule Y. After a few steps, we can arrive at a capsule that we don't yet have — a new capsule, a new idea. Of course, we took note of it, paused to explore it, and recorded it in our database of consciousness capsules — if it carried any meaning for us, if we had been looking for something of that kind.

Today, 60 years later, I do not remember any specific examples that I experienced. However, I remember very well the sensations they evoked, their style. So now I will invent an example that will be exactly identical in spirit to what I actually observed. This is necessary because it is very difficult to imagine reality, but it must be seen and felt in order to understand our consciousness.

By the way, I never managed to "capture" a chain longer than seven links, which, as I understood decades later, makes sense. In the second volume of Visions in Mathematics: Towards 2000 (GAFA 2000), this topic is explored in the section Discussions at the Dead Sea, in the discourse of Mathematics in the Real World.

In my talk at this published discussion, I pointed to one well-known fact. Our brain is unable to simultaneously control and keep in memory more than seven different events (thoughts). By the way, I believe this might be possible if the events are clustered in a very narrow domain and the person is specially trained in this domain — for example, recognizing various scents. Since most of the phenomena around us depend on a number of parameters much greater than seven, in order to tackle this problem, mathematics was created and is being created. This is the goal of mathematics — to enable our brain to oversee a much greater number of events than just seven. This is the goal of abstract thinking. I will not go into a discussion of this rich subject here.

By the way, this number may be different for different systems with consciousness. For example, experts told me that in dogs this number is four.

Again, there is an explanation for both the numbers seven, or more precisely, likely eight (it's just that one of them turns out to be redundant — an empty set in the system), and the number four. In the binary system, one of them has two digits and the other three.

However, it is interesting and surprising that this last observation is consistent with the property of our brain we discussed earlier — the number of episodes in a single impulse. Perhaps there may be many more of them, but our brain is "unable to tell" us that.

And now, an example of one impulse that I promised to give. Initial situation: I am at a conference, and in the morning at the hotel, I'm going down for breakfast. I see sliced pineapples.

1. First episode: I am at the war in Lebanon, 1982.

(The reason why this episode appeared: I was a soldier in Lebanon, and I really didn't like cooking for myself. This is why, when I opened a food package designed for 10 people and for more than one day, I would pick a large container of canned pineapples, open it, drink the juice, and snack on the pineapples. Now, whenever I see a pineapple, it is associated with the war in Lebanon).

2. Second episode (the analogy is obvious): Lebanon, I am lying under the car (the only place in the shade, while the air temperature is above 40 °C) and listening to a classical concert on the radio. Boris Berman, my friend of that time and a remarkable pianist, is performing.

3. Third episode: my thoughts skip to Piatetski-Shapiro, our mutual friend of the first years of emigration to Israel and an outstanding mathematician. We often discussed with Boris his achievements and his rise, since Boris really wanted to replicate that success in his music career.

4. Piatetski-Shapiro introduces me to another remarkable mathematician from Princeton, Elias Stein.

5. Now Princeton — my thoughts drift to Jean Bourgain, a brilliant mathematician who left us early and with whom I have many joint papers.

6. Now Jean Bourgain — some conversation somewhere about something. I do not remember.

7. But the thought catches mathematics of some sort, and that's it — now I'm in mathematics.

The impulse is over, and I am pondering new thoughts that appeared. All seven intermediate episodes have already been forgotten. I want to explain why I decided to also include the war in Lebanon in this invented example. The truth is that the only photographic snapshot that I remembered from my university time experiments was the following. This is a frozen snapshot of a horrific episode when one of our female students attempted suicide in the presence of several of us. She had an ampoule of potassium cyanide in her hands, and because of the tension, she accidentally crushed it. Luckily, she didn't cut herself, and everything turned out okay. It all ended well.

None of the people involved in this episode was in any way connected with either the previous photographic snapshot or the subsequent one. Some element in this snapshot connected with a previous episode, and some other element with a future episode, but I don't remember the details. This is why the war in Lebanon seemed to me the closest to that episode of all that happened to me later in life.

Additional Remarks on Consciousness

The additional remarks in this section concern only our consciousness, the consciousness of people.

I noted previously that early in my scientific pursuits, I performed some experiments on myself to check how my brain works. Of course, these can also be considered experiments on consciousness. One example I believe to be of interest is the following.

I noticed that working for 10–12 hours without a break from evening to morning puts the brain in a completely new state. Apparently, as in long-distance running, you get a "second wind", and the brain switches to a different mode. Whoever has not tried this and has not experienced this mode does not know the power of his brain, does not realize what a powerful tool he is endowed with. It wasn't often that I experienced this, and by the time I was 30, I was no longer physically capable of working in such an intense mode. It is difficult for me to describe this state of mind today, as too many years have passed.

Why does this happen? Perhaps, in a state of intense mental focus, the number of impulses of consciousness that I talked about in the previous section sharply increases. Additionally, the search for suitable analogies and their analysis also significantly expands. I should note that solutions to the problems that I thought about under such conditions do come. But invariably, in the morning, after rest, they turn out to be incorrect. But this is not so important, because a new understanding comes. The boundaries of the unknown shift. And this happened every time.

This observation echoes a similar observation I read about in Vladimir Arnold's work. But he, if this was in the winter, would take his cross-country skis and go on a long, extended run (during the day). Arnold (1937-2010), who was one of the most extraordinary mathematician of our time, was a "walrus" (that is, he could plunge into icy water in the winter), and he went skiing without outdoor clothing. He also wrote that during such a run he seemed to have solved the problem, but he was also sure that there was an error, which he found after resting. However, he came to understand something, and the boundaries of the unknown shifted.

I have observed other methods of inducing maximum mental focus and intensification of one's consciousness. For example, Gromov "plunged" into silence, absolute silence. I described this in my essay about him in the book *Encounters with Mathematicians*.

There is another amazing observation that I will share here. I take it from my other book, *My Non-Mathematical Biography*, the chapter titled "Observations on How Our Brain Works."

Those were very strange sensations that I developed during one very intensive and successful academic year toward spring. I began to sense the nearing solutions to the problems I was working on. Even before I knew the solution, I developed heart palpitations and some kind of a strange sensation inside: any moment now, it's almost here, just give it a push — somewhere in the subconscious, everything was already clear, and now I just needed to "receive" it, not let it slip away. And I was not mistaken — the solutions to the problems did come. I reckon that every two weeks that year, I had a new, non-trivial and often well-known problem solved.

I have no choice but to digress here to describe our, Israelis, state of mind during and immediately after the 1991 Gulf War, when the Scud missiles launched by Saddam Hussein, the ruler of Iraq, were falling on Israel. There is a certain similarity in the sensations, but the state of mind that we experienced during the war is understandable and easier to describe.

When the siren sounded, we had about 90 seconds to prepare for the missiles to fall. Everyone jumped up (this usually happened at night) and quickly performed their tasks, including our kids, still very young at the time: we shut off the gas and electricity, hermetically sealed off the one room in which we were (everyone was afraid of a chemical attack), put on gas masks, and covered our heads with the mattress (in the event of a very close strike, glass debris and objects from the ceiling and walls could fall on the head). Of course, there was a huge amount of adrenaline released into the bloodstream, but we did not feel it — the adrenaline was doing its work. Soon, the reflexes were so well-drilled that the siren of an ambulance (or a police car) somewhere during the day and completely unrelated to an attack caused the same reaction and a huge rush of adrenaline into the blood. However, it was no longer needed in this case, and we knew it immediately. And there was a reaction to the unused adrenaline, very unpleasant and severe: the heart was pounding, and everything inside seemed to drop and freeze. This is a dreadful sensation, and it takes a while to recover. In Israel during that

time, TV and radio programs muted the sound when there was a siren in the broadcast. Ambulances also tried to avoid using them.

Back to the subject, the sensations I experienced when the solution to a problem "left" the subconscious and passed into my conscious mind was somewhat similar. But instead of a sensation of something dropping inside, there was a feeling of anguish accompanied by heart palpitations. Perhaps some other chemicals were released into the bloodstream (or a smaller dose of that same adrenaline) and contributed both to this state and the process of transition from subconsciousness to consciousness. By the end of the summer, I got scared. I was afraid that my heart would give in, but I could not stop the onset of these sensations. My wife Ludmila remembers how I began to persuade myself that I no longer wanted to prove theorems, I did not want these sensations — I wanted to take a break from them. And, to my regret, they stopped after a couple of months.

A few years later, when the new mathematics of that year had already been "digested", I tried very, very hard to evoke those same sensations inside me, to renew "contact" with my subconscious (I am joking, but you never know), but it wasn't working. Only 20 years later, in the middle of the decade from 2000, I felt on several occasions that I was very close to it, but nothing got proved in those minutes, and the event was not completed. (This is how it is — we always want things we don't have, and when we have them, we get scared.)

I have another example of the connection between the subconscious and the body's "chemistry". This example comes from one of the most talented mathematicians of our time, Ofer Gabber (IHES). Unfortunately, he was an absolute perfectionist due to which Gabber hardly published any of his works and is thus only known within the circle of algebraists. But in that circle, he is treated with a nearly religious fervor. He often answers questions about problems that have eluded the best minds for years, and does so "off the hip", during lectures and seminars, and the whole of algebraic geometry has been moving ahead in the 1980s and 1990s under his influence. I am interested here only in one story.

Deligne, one of the greatest algebraists of our time, wrote a lengthy paper (about 200 pages), which was supposed to be a joint work with Gabber, and Gabber was supposed to read the text and offer his opinion and comments.

His perfectionism was delaying the publication of very important results, so Deligne was getting anxious and asked me to help. I had a conversation with Ofer. His position was that there were errors in different places throughout the paper, and therefore he could not agree to its publication. "But this is impossible," I said. "I cannot believe that you would point out to Deligne the errors, and he would refuse to fix them." "It's not that simple," Ofer replied. "They (the story dragged out for years and the paper acquired other coauthors) want to present everything at a level of abstraction where many details of the theory have never been properly verified and written down. I cannot point out where things don't quite work as written, but when I read an inaccurate or erroneous statement, I get a stomachache, and when I read this text, my stomach hurts all the time!"

I had no answer to that. The paper, which actually turned into a book of about 350 pages, was published without Gabber's co-authorship, although in the very first paragraph of the introduction it was written that the authors consider Gabber one of the co-authors of this paper, who, not being a mere mortal, could not take upon himself the burden of potential errors.

Thus, here we have a sign from the subconscious, a "stomachache" or, more likely, unpleasant sensations inside the body.

I know of, and have witnessed, more similar reactions from Gabber, when without pointing to the specifics, he would exclaim: "Stop! There is an error here." And he would be right. Once, in my presence, Gromov (who is one of the best World Geometers, if not the best) tried to explain to him a certain geometric point three times (Gabber was an algebraist, not a geometer), and was stopped by Gabber, who said that from a particular juncture he did not understand. This was Gromov's first meeting with Gabber. It happened when Gromov asked me to invite him to go to lunch together. As this conversation was happening, I could observe Gromov getting angry and could sense him thinking to himself: "what kind of an idiot has Vitali introduced me to." But the third time around, Gromov realized that he, Gromov, was mistaken! He also could not see how to continue from that juncture.

Concluding Remarks

I'd like to start by listing some of the conclusions that I think this text brings us to, and then discuss possible sequels. Different levels of consciousness accompany and coexist with any life, not just humans. Consciousness is an automatic function of life. However, more than that, any system that depends on a very large number of variables automatically has certain properties reminiscent of the function of consciousness. The mathematical model of these properties corresponds to the concentration phenomenon, which we observe in any system with a huge number of dimensions (with sufficiently rich symmetries). This similarity lies in the fact that when a certain life situation is repeated, consciousness helps us choose the response — the reaction (that is, the form of behavior) — even if the situation differs from the one that has already been tested once before, but does not differ too significantly. And this corresponds to the concentration phenomenon in mathematical models.

I will not elaborate here on the obvious point that this repetition of the reaction helps to sort out useful knowledge from useless knowledge (Darwinism, but at the level of selection of individuals with useful, "correct" consciousness). The level of thinking should be as standardized as the form of behavior.

We mainly discussed not consciousness as a whole, but its elementary unit — a conscious action. Consciousness as a whole is the aggregation of a huge number of conscious actions that a given subject (living being) possesses. Consciousness gradually emerges with the increase in the number of parameters on which this system depends.

However, the starting point may be an individual being, such as an ant or a bee, and as their numbers increase, slowly and gradually there arises the consciousness of the corresponding communities consisting of a very large number of individual members. This, of course, can also be applied to humans.

I use the term "hamula" (taken from Arabic) to define a very large group of subjects united by certain affiliation, but not yet large enough for the emergence of full-fledged consciousness. A preliminary stage, so to speak. For example, fans of some highly popular football club are already a hamula with a certain set of well-defined conscious forms of behavior. However, as it grows, a hamula can progress to the stage of a super-hamula — a new organism with its own stabilized consciousness. This is a situation where the existence (and life) of an individual member is completely subordinate to the group consciousness and is no longer viable individually (again, ants or bees).

Let us now suppose that these individual members were subordinate to certain rules of behavior, or to a certain, let's call him, Creator. This was their individual consciousness. But now that they have grown into a superhamula, all these rules have disappeared. They are now subordinate to a new consciousness, which, by the way, they do not control. This could turn out to be extremely dangerous for the creator of this supersystem.

This is why I think that, when building so-called thinking machines, or supercomputers, we should have tests in place for the emergence of "consciousness" and should destroy the system immediately if the tests are positive. Frankly speaking, I feel scared at the moment. Have we, the humanity, transformed into such a supersystem with a different consciousness, and are we not exposing ourselves to the danger of destruction due to the emergence of this consciousness?

Appendix A. Examples of Consciousness for Different Living Things

Animal World

I will talk now about a very intelligent bird, and once again we have come to the magpie, this time the Australian magpie. We were renting a small house on the outskirts of Canberra. There were always birds in the small area in front of the entrance. Of course, our children, nine-year-old Anat and 11-year-old Emanuel, fed the birds a little. There was even some kind of a small feeder on the ground a couple of meters from the entrance. We had already been living there for a while, and the birds were also used to us, although they scattered when we walked past them.



Australian Magpie

Once, fairly early in the morning, there was a small knock on the door. We were all downstairs already, not far from the entrance. Luda went to open the door. Our front door opened into an entrance hall, about two by two meters, which led into the living room through an open entryway. Luda opened the door and saw no one. She then looked down and saw a bird standing by the entrance! It was the bird who knocked on the door! We were all nearby but standing in the living room. Luda moved away. And the bird slowly, in no haste, came in and made a circle around the entrance hall. It was obvious that the bird was terribly nervous. It even relieved itself a little on the floor along the way, out of fear. But it made this journey and went back to the porch. None of us moved. The bird breathed a sigh of relief (this was my interpretation, but it was an obvious one). It then glanced at us again and jumped off the porch. The feat was over. It went to the feeder. And we realized that it showed all the birds that were watching from all sides that this was its place now. The bird claimed it and was now the master. And from that point on, all the other birds waited for it to finish its meal and move away from the feeder. And then the other birds could eat too.

Once again, we see a clearly set goal, a complex and risky plan, and its rigorous implementation. What can we learn from this story about this civilization, the civilization of Australian magpies?

Cells

One of the most incredible revelations of some 25–30 years ago was the discovery of APOPTOSIS. All of my knowledge on the subject comes from our discussions with Professor Elena Vladimirskaya (Lena). In short, apoptosis is the program inside a cell that is activated to kill it. That is, a cell may receive an order to die! This, of course, is a necessity based on the survival "instincts" of a multicellular body, e.g., such as protection from a quick cancer-related death that will most likely occur if the cells are allowed to undergo uncontrollable division. In fact, the regulation of how a cell dies should be very strict. The incredible thing is that such a program is actually continually active in a cell, always ready to act. The only way for a cell to continue living is to perform a certain function, which it is designed to keep performing. Then the action of apoptosis is delayed up until the cell starts performing something else.

By the way, there are two types of cell death. One is the long-known necrosis. In the event of such death, the membrane is destroyed at a very early stage in the process, and the death is always associated with external stimulation. Usually a large number of cells are involved in this process at the same time (we observe pus in this case).

The second type of death is apoptosis. In this case, the membrane is destroyed at the very end of the process, when all of its contents are, as it were, packed into the so-called apoptotic bodies that do not irritate the environment. Most often the process is genetically determined, although it can be triggered by some danger from the outside — for example, if a cell does not want to let in a virus that would multiply inside it and pose a mortal danger to other cells.

I will now illustrate how clever a cell is by demonstrating some of its actions. To be sure, I will simplify the reality. My apologies for this. On its surface, a cell has numerous receptors — their count is likely to be in the hundreds of thousands but could be below one hundred. To help us see the picture, let's imagine a cell rescaled to the size of a small town like Ramat Hasharon (where I was living recently). Then the receptors would be some structures the size of around three to five stories, on its side surface. (Let's remember that cells are three-dimensional, like a ball, and not two-dimensional, as we may perceive a city on the surface of the Earth.) Now let's imagine a molecule approaching the cell. It contains some information, which could be an order for the cell to carry out a certain action. It may enter the cell ONLY through these receptor structures (whether it is a physical intrusion or some message being delivered). The relative size of the molecule in this chosen scaling would be that of a person.



This is the corona virus; It is 100 times smaller than a cell, But its receptors are about the same size as those found on cells.

Not every receptor is ready to accept every messenger molecule — receptors are molecule-specific. And there may be no receptors at all on a given cell for some molecules. These molecules are not allowed inside and thus do not carry any information deliverable to this cell. But let's assume that there is a receptor, and a molecule has arrived that is suitable for this receptor. Now the receptor needs to make a decision as to whether or not to let it in (the molecule or the information).



It is a cell, and numerous receptors are visible on its surface.

Stop! Not so quickly. One single receptor will NEVER make the decision. It will either call a similar receptor located nearby, so that this receptor can move toward the original receptor, or it will create an identical receptor nearby (duplicate itself). These two receptors will then jointly make the decision.

Again, stop! Not so quickly. This scheme applies to certain "simple" decisions. If the molecule carries such a crucial order as to start apoptosis, then two receptors will not be considered good enough. Depending on the

form of apoptosis, at least three receptors will be needed, and possibly as many as six of them.

Roughly speaking, there are two types of apoptosis: a slow one, which lasts many hours and can be stopped and reversed during its development, and a very quick, immediate apoptosis, which cannot be stopped after it starts. The first, the slow one, is initiated inside the cell. It is exactly like this — an order to die inside the cell, to commit suicide, due to something that went wrong inside the cell (the risk of uncontrollable division is the greatest danger for the body, in which this cell is an integral part). This is a very interesting process, which I will explain, again, in a very simplified way.

There is a certain gene in the cell that "observes" the scene. If it notices that something is wrong, it immediately stops all activity of the cell, letting it repair the system and return to normality. If this does not happen within a certain period, this gene (the "night guard," as experts call it) activates a family of genes (which I will call the "jury," as it plays this role), 16 such genes altogether, of which 10 are always pro-apoptosis (let's denote them with a "+" sign), and six are against apoptosis (I will denote them with a "-" sign). These genes produce some molecules that are involved in certain activities that end in joining these molecules into pairs. There may be pairs of (+,+), (+,-), or (-,-) type, although some may remain single. What happens next is the counting of the "votes," whereby the "lonely" (unpaired) molecules are not counted (they "did not come to vote"), and nor are the "indecisive" (+,-) pairs (abstained votes), while the (+,+) pairs count as "votes" for, and the (-,-) pairs against, apoptosis.

The majority decides the fate of the cell. (If you ask me what happens if the votes are split evenly — well, I don't know! But I suspect it is the same as a no-apoptosis vote). One may notice a more substantial problem: the *a priori* prevalence of pro-apoptosis molecules, you might think, implies a predetermined pro-apoptosis decision. However, there is another parameter involved. These genes (the 16 genes of the "jury") produce molecules with varying ability to join another molecule from this family. Each of them has regions responsible for the ability to join another molecule. Among the six anti-apoptosis ("-" sign) molecules, four have four such regions, and the remaining two have three regions. However, among the 10 pro-apoptosis ("+" sign) molecules, only three have three regions (no molecule has four regions!), one has two such regions, and the remaining six each have only one such region!! Thus, they have a very weak ability to co-join. And this creates the balance!

Note that since the number of "+" molecules, that is, the pro-apoptosis molecules (they are protein molecules), is larger, if all 16 molecules pair up, then the answer will always be for apoptosis. But the overseeing gene, which I call the judge, and experts call the "night guard," counts the votes after a certain time. This gene-judge chooses the time. So the time it gives the protein molecules to come together is limited, and it is another parameter of the process. Now I'll suggest a conjecture. I have not read anywhere about what I will write now, but it seems to me absolutely natural and something that simply has not been tested. All molecules that are against apoptosis have a very high probability of connecting, since they have so many places where they can connect with other molecules. At the same time, molecules that are in favor of apoptosis are much less likely to combine with any other molecules, for or against apoptosis. Let's say four of them still have a more or less reasonable probability of connecting, but the remaining six have a very small probability. Therefore, if the gene-judge gives a short time for the decision, then most likely the decision will be against apoptosis. But if it gives a very long time, then the decision will almost certainly be for apoptosis.

So, how does the gene-judge choose the time? This is not known for certain, but I will return to this issue in a few lines.

If apoptosis is chosen, the other gene is activated, and a very interesting next step starts: the real "killer" gene is activated, which does the job (also very interesting). However (!!), there are some proteins that may block the action of the killer. For the whole body of cells, this is a very bad sign, since in this case the "unleashed" cell will start uncontrollable division (cancer). I will drop this part now. But what happens if the decision made is against apoptosis? Then the cell continues performing its job, the one it should be doing and was doing when it was stopped, and everything looks normal. However, our main "judge," the "night guard," may restart the process and call again for the above-described family of 16 to take another "vote" if it is still worried about something.

I return to the question of the time selected by the judge to count the votes. Of course, it is possible that some chemical substance is released into the space where the activity — the dance of molecules — occurs. I call it scent. And this scent affects the time period chosen by the gene-judge. But in actuality, this hypothesis is not needed to solve the problem at hand. The

judge can start from a short time period, and thus the decision will be against apoptosis. However, we have just explained that if the judge is still concerned about something, it will repeat the process. And then, the second time, the time for the molecules to dance will be longer, and the probability of making a decision for apoptosis will be higher. Finally, if a decision against apoptosis is still made, and, after the cell returns to normal operation, the gene-judge becomes worried again, then it will start the process again, and the third time, the time interval selected by the gene-judge can be very long, and a decision for apoptosis will be made.

Wouldn't you agree that cells act in a much cleverer way than we, humans, do?

I think I should feel pity for you if you still don't recognize "consciousness" in such behavior.

I apologize for explaining this in such a primitive manner. But it is impossible to explain this more precisely and in greater detail in a short nonscientific essay. I just wanted to point out how well-reasoned a cell's actions are.

"Thinking" Trees

Now I would like to change the "scale" of our discussion and discuss the "consciousness"/"thinking" of some huge living objects. I will talk about trees. Of course, in general, trees are so different as a species that we are unable to recognize their traces of "thinking" or "consciousness." But there are two very different kinds of trees that both show signs we can recognize. I personally had a chance to observe both of these kinds, one in the jungle of Amazon, and another in Cambodia. Perhaps there are many more, but I have only observed these two.

One of these kinds is a "walking tree." These are trees that move (walk) along the Earth surface. Not quickly — say, around five meters per year. Whoever did not know this and does not believe me, search Google for "walking tree" and see hundreds of pictures, including those that show the very process of this "walk." By the way, these are very big trees, with large trunks, that reach high into the sky. Around two meters from the ground, such a trunk is divided into lots of "branches" going down. One may think of them as roots that hang a couple of meters above the ground level. To move in a specific direction, the tree sends from its trunk above the ground new roots in the chosen direction, and when these roots firmly settle inside the soil, some roots on the opposite side, the ones no longer needed, die, and hence the whole trunk (and thus the tree) shifts in that direction. Think what kind of coordination needs to be in place in order for the trunk to remain stable and "looking up," not falling! And where is that "brain" that regulates this? Thus, our belief that a brain is absolutely needed to "compute" and decide how to perform some clever action is wrong. This can be done without a brain. Making the choice of direction in which to move may be easier to explain.

I have read that perhaps the tree needs more sun. These trees live in the tropical jungle of South America. At least this is where I saw them, and the direction in which the tree chose to move was obvious. From what I saw, it looked like the tree was trying to escape falling into a deep ravine, perhaps the result of the recent rain season.





The second kind of "thinking trees" we encountered in Cambodia were the so-called giant Strangler Figs. Those are huge, enormously big trees, living around a thousand years or more, which completely covered, fully destroyed the old cities of Cambodia. People here believe that without any war, entire citadels and residencies of very powerful kings were suddenly abandoned, and the nation moved away to build new citadels somewhere else.

These trees "eat" other trees. I witnessed this. The process starts with what looks like harmless lianas using the trunks of other trees for support and climbing around them and skyward. When such a "liana" firmly establishes itself around a neighboring trunk, it starts joining with other "lianas" to turn into a single formation that becomes another trunk around the trunk of this tree's "dinner tree." After a while, it is all over for the "dinner tree," as it gets strangled and eaten. I think that at the previous stages of its growth it obtained nutrition from the tree it invaded. I have photos of all the stages of this process. But these giant trees have also destroyed buildings and huge

structures through a more complicated process. However, I have difficulty explaining this in writing (see photos: on the top left photo you can see inside the trunk another eaten tree).



I was amazed at the way these trees reproduce. Of course, they can also germinate from seeds fallen to the ground. But this growth process would take many years, perhaps decades. But what difference does this make for trees that are going to live for about 1000 years? So what is the reason for their preferred, as we can observe, method of propagation?

When its seeds fall on another tree, or perhaps also on stones of castles and fortresses, they sprout and twine around the tree like harmless lianas. They feed on this tree, and once they are wrapped around it fairly tightly, they merge into a single trunk! Their roots descend, reach the ground, and enter the soil. And at once, you have a large tree. This process lasts no more than a year. Now it will sprout, grow, and become a giant.

Why might this be necessary? If, for example, for some reason a tree needs to appear very quickly. If the growth came simply from the soil, it would take decades for the tree to reach this state. By the way, seeds can also attach themselves to stones from which citadels are built in Cambodia. One factor that has helped the growth of the strangler fig trees on walls of the buildings is the unique property of the stones used for building the temples. Angkor temples are made of sandstone/laterite which is porous in nature. This enables the roots to extract water from the stones.



Stones start cracking in the grip of giant roots of Strangler Fig trees – look at the grip of the root. Photo:Indrani Ghose.

These trees begin as epiphytes (as plants that grow on other plants)

And another example:



New roots wedge through the gaps of the building blocks. Photo:Indrani Ghose.

We also saw these trees in the forest, far from man-made structures. Those were large trees, but they were large in a "normal" way. However, to destroy buildings and fortresses, they turn into the giants you see in the photographs. The only thought that comes to mind is that these trees are the protectors of the forest. Human settlements destroy the forest, and when this happens, these trees destroy the settlements. We know that the ancient peoples of Central America abandoned their developed settlements after a certain number of years and moved away to build new ones. The reasons for this are unknown to science. Perhaps they saw signs that the forest was starting to drive them away.

Appendix B. References to Some Mathematical Background

I will provide references here to some mathematical papers where one can read at the professional level about some of the ideas used in the main text.

For this book, the following reference is of primary importance (Section 4 of the paper):

V.D. Milman, Simplicity v/s Complexity in the Framework of Geometric Asymptotic Analysis and Some New Applications of the Concentration Phenomenon, Milan j. Math. 74 (2006), 199–211.

The text of this paper is published on my Home Page.

In addition, to understand the stability of our model, which we discuss at the end of the section The Mathematical Model of Consciousness, the following article by Shiri Artstein-Avidan is important:

<u>Artstein, Shiri,</u> Proportional Concentration Phenomena on the Sphere. *Israel J. Math.* <u>132 (2002)</u>, 337–358.

This paper provides computation of the measure on the unit sphere in very high dimension of small-dimensional subspheres, as sections of the original sphere. It is based on her computations that in our discussions we could consider subspaces even of dimension o(n) instead of sqrt(n), and, in any case, changes of our function on such subspaces would not have essential influence on the averaging value of the original function.

The idea of Concentration Phenomenon as a general method was put forward in the paper

<u>Milman, V. D.</u> A new proof of A. Dvoretzky's theorem on cross-sections of convex bodies. (Russian) *Funkcional. Anal. i Priložen.* **5** (1971), No. 4, 28–37

and was continued in two articles in <u>Dokl. Akad. Nauk SSSR</u> [Proceedings of the USSR Academy of Sciences] (translated into English in the Soviet Mathematics—Doklady translation journal).

Here is one of them:

<u>Milman, V.D.</u> Asymptotic properties of functions of several variables that are defined on homogeneous spaces. <u>Soviet Mathematics—Doklady</u> **12** (1971), 1277–1281.

A comprehensive discussion on this subject can be found in:

M. Ledoux, The concentration of measure phenomenon. Mathematical Surveys and Monographs, 89, American Mathematical Society, Providence, RI, 2001.

Historical remarks, known methods for detecting the concentration phenomenon (as of the year of publication of the article), and numerous examples of the application of this method in mathematics are presented in:

V.D. Milman, The heritage of P. Levy in geometrical functional analysis. Colloque Paul Levy sur les Processus Stochastiques (Palaiseau, 1987). Asterisque No. 157–158 (1988), 273–301.

The following review article may also be useful for sensing mathematical objects in very high dimensions:

V.D. Milman, Phenomena that occur in high dimensions. (Russian) Uspekhi Mat. Nauk 59 (2004), No. 1(355), 157–168; translation in Russian Mathematical Surveys 59 (2004), No. 1, 159–169.

Side Story

A Note on Fields Related to Consciousness.

In exploring human consciousness, it would be extremely interesting to study its development, the progress in our consciousness, and also the extent to which it influences our behavior, how it gets us "entangled" when it comes to our freedom of choice. This is a completely different question, independent from the one we dealt with in the main text. It does not depend on what consciousness is and how it arises. I will outline below three areas where progress looks possible. These are just a few trivial observations, but serious research in these areas could prove extremely non-trivial and fascinating.

"Archeology" of Consciousness

It would be very interesting to have information about the evolution and progress in the development of consciousness. A priori, this seems utterly impossible, since there are no physical traces of the consciousness of people of the past. However, this is not the case, since we see and observe the development of language, and it reflects the development of consciousness. I will give one small, but I believe rather striking, example.

I will now consider the etymology of cardinal and ordinal numbers in several languages.

English:

One, two, three, four... (cardinal numbers) First, second, third, fourth... (ordinal numbers)

Note the completely different roots in the first two numbers and the emergence of a pattern starting with the third number.

R

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Фдин, два, три, четыре...

фотописiation: odin, dva, tri, chetyre...)

в

Первый, второй, третий, четвёртый....

фoronunciation: pervyi, vtoroi, tretiy, chetvertyi...)

(brdinal numbers)

i

We observe the same phenomenon — different roots in the first two numbers,

and the emergence of a pattern starting with the third number.

I

The same picture in Hebrew:

ehad (1), shtaim (2), shalosh (3), arba (4), hamesh (5)...

(pardinal numbers)

bishon, sheni, shlishi, revii, hameshi...

(ordinal numbers).

r

Because of the Hebrew reading rules, one might think that arba (four) and
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Because of the Hebrew reading rules, one might think that arba (four) and revii (fourth) have different roots. But this is not the case. It's just that the same letter can be read as "b" or as "v".

We can conclude that there is a huge gap in time and interpretation between the numbers 1 and 2 and those that follow — 3, 4, and so on. For two numbers, there's a simple system of interpretation: yes and no; either something exists, or it does not. And for that reason, their use to refer to quantity and sequential order was completely different. However, it turns out that the next step after them, the number three, already represented such an abstraction that there was a leap of consciousness, and all the names of subsequent numbers were already created according to a fixed pattern connecting the number as a quantity and its sequential order. I think that language experts could provide many great examples and thereby outline the stages of development of consciousness.

Does freedom of choice (free will) exist?

This question is not as simple as it may seem, and it is closely related to our consciousness. Indeed, in our mind, in our thoughts, in our dreams, we have unlimited freedom of choice. But we need to understand that there is a difference between what we know how to do and what we would be able to do. Here's an example to explain what I mean. Naturally, I know how to undress. I go to bed every day, take a shower, and undress every time. Does this mean that I can undress and go naked somewhere on the street or in some square? No, absolutely not. I would not be able to do this. There is a difference between what we know how to do and what we can do. Thus, I don't have the freedom of choice to undress on the street, even though I know how to do it.

We rarely engage in the analysis as to what we would be able to do of the things that we know how to do. And even when making very serious decisions, we do not always analyze the restrictions that we are subject to, either from the outside or from within us. But we are entangled in these restrictions, and we don't realize to what extent.

Again, sometime in my early years, somewhere before age thirty, I would walk mentally through my life and try to assess whether I had the free will to make a decision different from the one I had made. What actually amazed me was that time after time the answer was No. I simply could not have made a different decision. I remember today that there was only one single case where I could not remember everything that was happening around me so as to understand whether or not I had another choice. Thus, it happened no more than once, and most likely not even once, that I had the freedom of choice and could have made a different decision.

(Of course, I am talking about serious decisions that affect our lives, and not something trivial, such as whether to go around a round table from the left or from the right, if both paths are clear and the decision does not affect anything later.)

Thus, the existence of freedom of choice remains a question for me. Perhaps our consciousness controls everything in a uniquely predetermined way.

Monozygotic Twins (Identical Twins)

What do identical twins have to do with the discussion about consciousness? Naturally, the question is: to what extent is consciousness hereditary? Studies of identical twins show that although there is an influence, it does not seem to be too significant. However, it is not only purely genetic material that subsequently determines development as a child and as a person. The initial cell carries a lot of additional information. And even for identical twins, this information turns out to be not identical. Thus, the question of how much our consciousness is determined by that initial cell from which we originated remains somewhat ambiguous.

I think that the closer identical twins are in physical appearance, the more identical those initial cells from which they originated must have been. This is why I would like to see the level of correlation between the physical resemblance of already grown adult individuals and the form of their consciousness.

During World War II and the very difficult times that followed, there were many cases where identical twins were placed with different families almost immediately after birth. Decades later, many of them found each other, so there were opportunities to compare their habits and biographies. There were many remarkable similarities. I will cite one case that has become a classic, although I remain relatively unmoved by this particular case. I think that without establishing the correlation that I spoke about above, it is impossible to conclude how strongly the genetics affects consciousness.

The classic case I mentioned is the story of Jim Lewis and Jim Springer, known as the "Jim Twins." They were separated at birth and adopted by different families who were unaware of the other twin's existence. Despite growing up in different environments, both Jims had strikingly similar life paths. The similarity of their life paths is remarkable, and even one such case speaks volumes. However, I don't know how similar their consciousness was. The researchers were more interested in the story and the similarity of their life paths than studying the similarity of their thinking and consciousness.